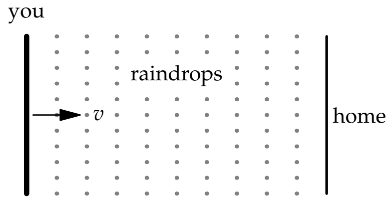
Instead, we invoked the 2-second following rule. As long as drivers obey it, the separation between cars equals 2 seconds of driving. Therefore, one car flows by every 2 seconds—which is the lane's carrying capacity (in cars per time). By finding an invariant, we simplified a complex, changing process. *When there is change, look for what does not change!* (This wisdom is from Arthur Engel's *Problem-Solving Strategies* [12].)

3.1.1 To run or walk in the rain?

We'll practice with this tool by deciding whether to run or walk in the rain. It's pouring, your umbrella is sitting at home, and home lies a few hundred meters away.

▶ *To minimize how wet you become, should you run or walk?*



Let's answer this question with three simplifications. First, assume that there is no wind, so the rain is falling vertically. Second, assume that the rain is steady. Third, assume that you are a thin sheet: You have zero thickness along the direction toward your house (this approximation was more valid in my youth). Equivalently, your head is protected by a waterproof cap, so you do not care whether raindrops hit your head. You try to minimize just the amount of water hitting your front.

Your only degree of freedom—the only parameter that you get to choose—is your speed. A high speed leaves you in the rain for less time. However, it also makes the rain come at you more directly (more horizontally). But what remains constant, independent of your speed, is the volume of air that you sweep out. Because the rain is steady, that volume contains a fixed number of raindrops, independent of your speed. Only these raindrops hit your front. Therefore, you get equally wet, no matter your speed.

This surprising conclusion is another application of the principle that when there is change, look for what does not change. Here, we could change our speed by choosing to walk or run. Yet no matter what our speed, we sweep out the same volume of air—our invariant.

Because the conclusion of this invariance analysis, that it makes no difference whether you walk or run, is surprising, you might still harbor a nagging doubt. Surely running in the rain, which we do almost as a reflex, provide some advantage over a leisurely stroll.

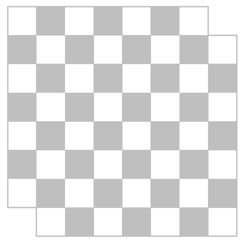
▶ *Is it irrational to run to avoid getting wet?*

If you are infinitely thin, and are just a rectangle moving in the rain, then the preceding analysis applies: Whether you run or walk, your front will absorb the same number of raindrops. But most of us have a thickness, and the number of drops landing on our head depends on our speed. If your head is exposed and you care how many drops land on your head, then you should run. But if your head is covered, feel free to save your energy and enjoy the stroll. Running won't keep you any dryer.

3.1.2 Tiling a mouse-eaten chessboard

Often a good way to practice a new tool is on a mathematical problem. Then we do not add the complexity of the physical world to the problem of learning a new tool. Here, therefore, is a mathematical problem: a solitaire game.

A mouse comes and eats two diagonally opposite corners out of a standard, 8×8 chessboard. We have a box of rectangular, 2×1 dominoes.



▶ *Can these dominoes tile the mouse-eaten chessboard? In other words, can we lay down the dominoes to cover every square exactly once (with no empty squares and no overlapping dominoes)?*

Placing a domino on the board is one move in this solitaire game. For each move, you choose where to place the domino—so you have many choices at each move. The number of possible move sequences grows rapidly. Instead of examining all these sequences, we'll look for an invariant: a quantity unchanged by any move of the game.

Because each domino covers one white square and one black square, the following quantity I is invariant (remains fixed):

$$I = \text{uncovered black squares} - \text{uncovered white squares}. \tag{3.1}$$

On a regular chess board, with 32 white squares and 32 black squares, the initial position has $I = 0$. The nibbled board has two fewer black squares, so I starts at $30 - 32 = -2$. In the winning position, all squares are covered, so $I = 0$. Because I is invariant, we cannot win: The dominoes cannot tile the nibbled board.

Each move in this game changes the chessboard. By finding what does not change, an invariant, we simplified the analysis.