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## EVALUATION OF LIMITING DRAWING RATIO (LDR) IN DEEP DRAWING PROCESS

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### Abstract

Part production by deep drawing technology brings important economic advantages. Cupping test is used to determine material suitability for deep drawing. The main principle of the test is redrawing of cylinder metal test piece to the cup. The result of the test is calculation of limiting drawing ratio (LDR) which states the ratio between the largest blank diameter and final cup diameter. Many cupping tests for various materials were performed in order to determine maximal value of LDR that would still allow deep drawing without failure of material integrity. Paper presents the theoretical determination of the limiting drawing ratio (LDR) in dependence on  $d_0/t_0$  ratio and friction coefficient between blank and punch. Limiting drawing ratio (LDR) converges to Euler's number ( $e$ ) for high test piece depth and final cup diameter ratio values. Theoretical analysis of the cupping on drawing die with tractrix curve shows that maximal value of LDR may achieve value  $LDR > e$ . This is possible at lower values of test piece depth to final cup diameter ratio. Coefficient of friction has determining influence on LDR value.

**Keywords:** deep drawing, differential equations, relative radial stress, cup, limiting drawing ratio, LDR

### 1 Introduction

Deep drawing technology is used to produce metal parts from various materials such as steel, copper, aluminum and their alloys [1-3]. The technology is used to make car body parts in automotive industry, cartridges in armament industry, mining blasting cups and various boxes and complex shaped parts [4]. Materials given for deep drawing must be tested for capability to be deep drawn and to determine limit condition for deep drawing [5, 6]. Testing is performed by the cupping test. Geometrical scheme of cupping process is shown in **Fig.1**. Cupping is performed without blank holder by two tools – drawing die and punch. Cylinder metal piece (roundel) with diameter  $D_0$  and thickness  $t_0$  is drawn by punch with diameter  $d_0$ . Outer diameter of final cup  $d_k$  is determined by the formula

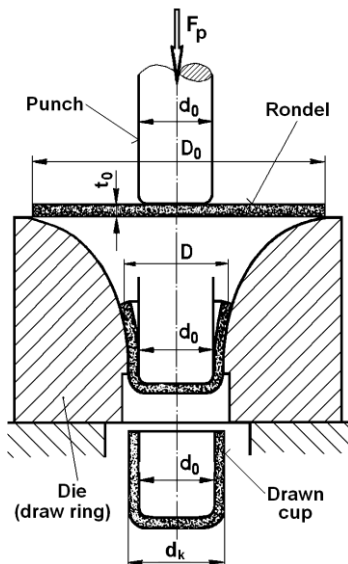
$$d_k = d_0 + 2t_0 \quad (1)$$

The thickness of material is not changing during the cupping process. The cupping test is characterized by LDR designed as  $\beta$  (non-dimensional value)

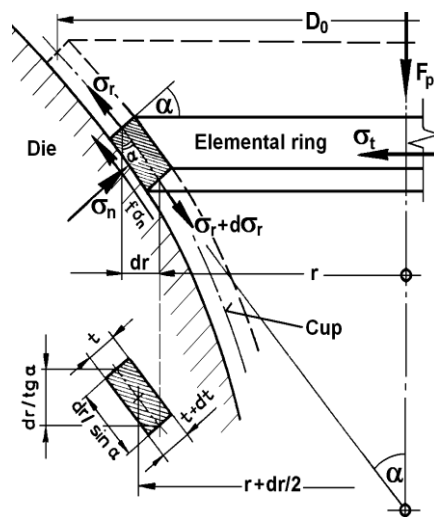
$$\beta = \frac{D_0}{d_0} \quad (2.)$$

That describes the ratio between the largest blank diameter  $D_0$  and drawing punch diameter  $d_0$ . Every forming technology has some material and technological limitation. If these limits are overrun, it leads to material failure [7, 8]. That means crack creation or even material destruction (breakaway of cup bottom) during cupping process. Therefore we are trying to find limits that would guarantee technological process without failure. It results from the formula (2) that the material has higher ability for deep drawing with increasing values of LDR. Many cupping tests were performed in laboratories to determine maximal value of limiting drawing ratio (LDR) designed as  $\beta_{0max}$  [9]

$$\beta_{0max} = \frac{D_{0,max}}{d_0} \quad (3.)$$



**Fig. 1** Scheme of the cup deepdrawing process



**Fig. 2** Annular element – Deep drawing process without blankholder

where  $D_{0,max}$  is maximal diameter of blank (roundel) at which the drawing cup is not yet damaged. Tests are performed in the step where the roundel diameter  $D_0$  is gradually increased until the cup breaks down. Maximal value of limiting drawing ratio  $\beta_{0max}$  warns about technological limits of cupping; and overrun of the LDR may lead to technological failure [1]. Generic annular element with marked stress effecting particular areas was chosen to define the state of stress (see **Fig.2**). Cupping is performed without the blank holder. Designation of particular stress is as follows: radial stress  $\sigma_r$ , tangential stress  $\sigma_t$  and normal stress  $\sigma_n$ . According to Shawki [9], the balance formula for forces influencing annular element (**Fig.2**) is

$$\frac{d\sigma_r}{dr} + f \cdot \left( \frac{d\alpha}{dr} - \frac{\cotg \alpha}{r} \right) \cdot \sigma_r = -(1 + f \cdot \cotg \alpha) \cdot \frac{\sigma_p}{r} \quad (4.)$$

where the geometrical position of annular element is defined by its radius  $r$  and the angle  $r$ . The deformation resistance is expressed by stress  $\sigma_p$  and friction between drawing material and die by friction coefficient  $f$ . The differential equation (4) is based on the condition that the wall thickness of a cup is not changing in the cupping process. The wall thickness may be considered as constant because of differential of the wall thickness that is equal to zero ( $dt=0$ ). This assumption allows to define  $t=t_0$ . The differential equation (4) is constructed with using of plasticity equation

$$\sigma_r - \sigma_t = \sigma_p \quad (5.)$$

Construction of the equation (4) in more detail with the commentary was published by Pernis [10]. Cupping material is expressed as  $\sigma_p$  (yield point of the material) in the differential equation. We exclude this material constant ( $\sigma_p$ ) from differential equation (4) and substitute it with relative radial stress  $\bar{\sigma}_r$  (6) due to general use of the equation [11]

$$\bar{\sigma}_r = \frac{\sigma_r}{\sigma_p} \quad (6.)$$

Differential equation (4) after the substitution (6) is as follows

$$\frac{d\bar{\sigma}_r}{dr} + f \cdot \left( \frac{d\alpha}{dr} - \frac{\cotg \alpha}{r} \right) \cdot \bar{\sigma}_r = -\frac{1 + f \cdot \cotg \alpha}{r} \quad (7.)$$

The relative radial stress is given by the equation (7) as a function  $\bar{\sigma}_r = \bar{\sigma}_r(\alpha, r)_f$ , where the friction coefficient is a constant. Geometrical shape of drawing die profile curve is generally defined as a function  $g(\alpha, r)$  [12]. It is necessary to define this function to find out the solution of the differential equation (7). Various curves or their parts were used in history of the cupping: line, sinusoid, circle, ellipse or tractrix curve. Real tests and measurements show that evolvent catenary curve known as tractrix is the optimal drawing curve. Therefore the tractrix curve is used to solve the differential equation (7).

## 2 Experimental Material and Methods

This paper is aimed at materials that are processed by deep drawing technology. These materials include the group of steel materials DC01 and DC06 of the field EN10130/91; and non-ferrous metals such as copper, brass CuZn30 and selected aluminum alloys. General differential equation (7) describing cupping without blank holder has no analytical solution. To get a specific solution of differential equation (7), the geometrical shape of drawing die with tractrix curve was applied. Analytical solution in the first step of differential equation was used and numerical solution was used for the second step. The evaluation of LDR was determined for relative radial stress that must meet the condition  $\bar{\sigma}_r \leq 1$ .

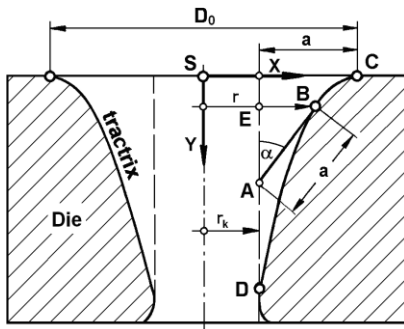
## 3 Results and Discussion

### 3.1 Drawing die with tractrix curve

The tractrix curve is defined as evolvent upon general catenary curve and it is described by the function (8)

$$x = a \cdot \sin t, \quad y = a \cdot (t - \operatorname{tgh} t) \quad (8.)$$

where  $t$  is a parameter. Drawing die with tractrix curve is shown in **Fig.3**. The curve starts at point C, then continues to point B and ends at point D. Zero point of coordinate system lies at point S. Parameter  $a$  describes the length of tangent to tractrix curve. The length of tangent is considered as the length from apposing point B on tractrix curve to point A. The value of angle is  $\alpha = \pi/2$  at point C and  $\alpha = 0$  at point D. The annular element (see **Fig.2**) is located at point B. The location of point B is defined by radius  $r$  and angle  $\alpha$ . Their correlation is defined by rectangular triangle ABE (see **Fig. 3**) where parameter  $a$  is given by hypotenuse AB. The radius of tractrix curve  $r$  for point B is given by addition of cup external radius  $r_k$  and triangle cathetus EB



**Fig. 3** Drawing die with tractrix curve

$$r = r_k + a \cdot \sin \alpha \quad (9.)$$

Where  $r_k$  and  $a$  are geometrical constants. Tractrix curve parameter  $a$  is defined by roundel dimensions and drawing punch diameter

$$a = \frac{D_0 - d_0}{2} - t_0 \quad (10.)$$

Differential of the radius  $dr$  is determined by derivation of the equation (10) with respect to the angle  $\alpha$ .

$$dr = a \cdot \cos \alpha \cdot d\alpha \quad (11.)$$

Radius  $r$  (9) and its differential  $dr$  (11) are then both inserted into differential equation (7) where the result differential equation (12) is

$$\frac{d\bar{\sigma}_r}{d\alpha} - f \cdot \left( a \cdot \cos \alpha \cdot \frac{\cotg \alpha}{r_k + a \cdot \sin \alpha} - 1 \right) \cdot \bar{\sigma}_r = -a \cdot \cos \alpha \cdot \frac{1 + f \cdot \cotg \alpha}{r_k + a \cdot \sin \alpha} \quad (12.)$$

There are two geometrical constants  $a$  and  $r_k$  in the equation (12). The substitution is used to reduce these constants to one non-dimensional constant  $k$ .

$$k = \frac{r_k}{a} \quad (13.)$$

It is necessary to express  $k$  constant through  $t_0/d_0$  ratio and also limiting drawing ratio  $\beta$  before the substitution.

$$k = \frac{1 + 2 \frac{t_0}{d_0}}{\beta - 1 - 2 \frac{t_0}{d_0}} \quad (14.)$$

Final differential equation suitable for evaluating is made by inserting the  $k$  constant (14) into the differential equation (12).

$$\frac{d\bar{\sigma}_r}{d\alpha} - f \cdot \left( \cos \alpha \cdot \frac{\cot g \alpha}{k + \sin \alpha} - 1 \right) \cdot \bar{\sigma}_r = -\cos \alpha \cdot \frac{1 + f \cdot \cot g \alpha}{k + \sin \alpha} \quad (15.)$$

The final equation (15) belongs to a group of differential equations that could be described by general expression

$$\frac{d\bar{\sigma}_r}{d\alpha} + P(\alpha) \cdot \bar{\sigma}_r = Q(\alpha) \quad (16.)$$

Where  $P(\alpha)$  a  $Q(\alpha)$  are known functions defined by equation (15) as

$$P(\alpha) = -f \cdot \left( \frac{\cos \alpha \cdot \cot g \alpha}{k + \sin \alpha} - 1 \right) \quad (17.)$$

$$Q(\alpha) = -\cos \alpha \cdot \frac{1 + f \cdot \cot g \alpha}{k + \sin \alpha} \quad (18.)$$

The method of constant variation is used for analytical evaluation of the differential equation (16) with regard to its type. The result of the evaluation is the expression

$$\bar{\sigma}_r = e^{-F(\alpha)} \cdot \left[ C + \int Q(\alpha) \cdot e^{F(\alpha)} \cdot d\alpha \right] \quad (19.)$$

Where  $C$  is integral constant (determined from initial conditions) and function  $F(\alpha)$  is expressed by integral

$$F(\alpha) = \int P(\alpha) \cdot d\alpha = -f \cdot \int \left( \frac{\cos \alpha \cdot \cot g \alpha}{k + \sin \alpha} - 1 \right) \cdot d\alpha \quad (20.)$$

The evaluation of the integral (20) depends on the value of constant  $k$ . There are considerable three cases of function  $F(\alpha)$  for deep drawing process [13,14]:

a)  $0 < k < 1$

$$F(\alpha) = -f \cdot \left( \frac{1}{k} \ln \left| \operatorname{tg} \left( \frac{\alpha}{2} \right) \right| - \frac{1}{k} \cdot \sqrt{1-k^2} \cdot \ln \left( \frac{1 - \sqrt{1-k^2} + k \cdot \operatorname{tg} \left( \frac{\alpha}{2} \right)}{1 + \sqrt{1-k^2} + k \cdot \operatorname{tg} \left( \frac{\alpha}{2} \right)} \right) - 2\alpha \right) \quad (21.)$$

b)  $k = 1$

$$F(\alpha) = -f \cdot \left( \ln \left| \operatorname{tg} \left( \frac{\alpha}{2} \right) \right| - 2\alpha \right) \quad (22.)$$

c)  $k > 1$

$$F(\alpha) = -f \cdot \left( \frac{1}{k} \ln \left| \operatorname{tg} \left( \frac{\alpha}{2} \right) \right| + \frac{2}{k} \cdot \sqrt{k^2 - 1} \cdot \operatorname{arctg} \left( \frac{1 + k \cdot \operatorname{tg} \left( \frac{\alpha}{2} \right)}{\sqrt{k^2 - 1}} \right) - 2\alpha \right) \quad (23.)$$

Integral constant  $C$  is determined according to **Fig. 3** for angle  $\alpha = \pi/2$  and relative radial stress  $\bar{\sigma}_r = 0$ . The value of integral constant  $C$  determined from (19) with respect to this condition is

$$C = 0 \quad (24.)$$

Specific value of relative radial stress  $\bar{\sigma}_r(\alpha)$  for the value of angle  $\alpha$  is determined from

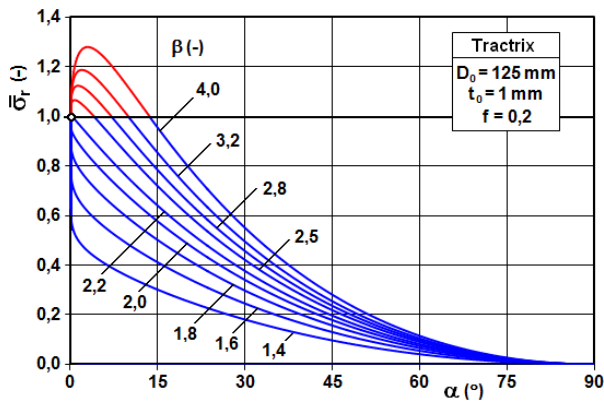
$$\bar{\sigma}_r(\alpha) = -e^{-F(\alpha)} \cdot \int_{\alpha}^{\pi/2} Q(\varphi) \cdot e^{F(\varphi)} \cdot d\varphi \quad (25.)$$

Numerical integration is then used to evaluate definite integral in the equation (25) for  $\alpha > 0$  [15]. The functions  $F(\alpha)$  and  $Q(\alpha)$  are in convergence to infinity for  $\alpha \rightarrow 0$ . Therefore the evaluation of relative radial stress  $\bar{\sigma}_r(0)$  from equation (25) is realized as limit value for  $\alpha \rightarrow 0$  [16]

$$\begin{aligned} \bar{\sigma}_r(0) &= \lim_{\alpha \rightarrow 0} -e^{-F(\alpha)} \cdot \int_{\alpha}^{\pi/2} Q(\varphi) \cdot e^{F(\varphi)} \cdot d\varphi = \lim_{\alpha \rightarrow 0} -\frac{\int_{\alpha}^{\pi/2} Q(\varphi) \cdot e^{F(\varphi)} \cdot d\varphi}{e^{F(\alpha)}} = \lim_{\alpha \rightarrow 0} \frac{Q(\alpha) \cdot e^{F(\alpha)}}{F'(\alpha) \cdot e^{F(\alpha)}} = \\ &= \lim_{\alpha \rightarrow 0} \frac{Q(\alpha)}{F'(\alpha)} = \lim_{\alpha \rightarrow 0} \frac{Q(\alpha)}{P(\alpha)} = \lim_{\alpha \rightarrow 0} \frac{-\cos \alpha \cdot \frac{1+f \cdot \cot g \alpha}{k + \sin \alpha}}{-f \cdot \left( \frac{\cos \alpha \cdot \cot g \alpha}{k + \sin \alpha} - 1 \right)} = \\ &= \lim_{\alpha \rightarrow 0} \frac{\cos \alpha \cdot (1+f \cdot \cot g \alpha)}{f \cdot (\cos \alpha \cdot \cot g \alpha - k - \sin \alpha)} = \lim_{\alpha \rightarrow 0} \frac{\cos \alpha \cdot (\sin \alpha + f \cdot \cos \alpha)}{f \cdot (\cos^2 \alpha - k \cdot \sin \alpha - \sin^2 \alpha)} = 1 \end{aligned} \quad (26.)$$

The evaluation of relative radial stress  $\bar{\sigma}_r(0)$  is described by sequence in (26). The relative radial stress at point D in **Fig. 3** is  $\bar{\sigma}_r(0) = 1$  and it is independent of constant  $k$  and friction

coefficient  $f$ . The absolute value of radial stress at point D is equal to deformation resistance  $\sigma_r(0) = \sigma_p$ . The equation (25) is visualized in **Fig. 4**. The distribution of relative radial stress is valid for friction coefficient  $f=0.2$  and initial dimensions of roundel  $D_0=125$  mm and a  $t_0=1$  mm. The parameter for particular curves is limiting drawing ratio  $\beta$ . All solutions of differential equation (12) for any constants run through point  $[\bar{\sigma}_r; \alpha] \equiv [1; 0]$ . Radial stress has a character of tensile strength; therefore, it cannot exceed the value of deformation resistance in deep drawing process. The curves where relative radial stress is  $\bar{\sigma}_r > 1$  make conditions for destruction of the cup in deep drawing process. The values of relative radial stress have to be  $\bar{\sigma}_r \leq 1$  in entire range  $0 \leq \varphi \leq \pi/2$  to avoid the destruction of drawn material. It results from **Fig. 4** that the value of limiting drawing ratio is  $\beta \leq 2.2$ . The relative radial stress  $\bar{\sigma}_r$  cannot exceed the value  $\bar{\sigma}_r = 1$  in entire high range along the drawing die.



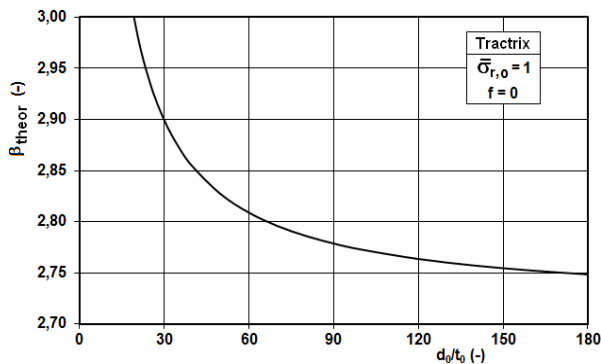
**Fig. 4** Distribution of relative radial stress along the high of cupping die

### 3.2 Limit values of LDR

The highest value of LDR could be reached from the equation (15) for friction coefficient  $f=0$

$$\frac{d\bar{\sigma}_{r,0}}{d\alpha} = \frac{\cos \alpha}{k + \sin \alpha} \quad (27.)$$

Differential equation (25) could be directly integrated. The value of relative radial stress  $\bar{\sigma}_{r,0}$  is evaluated at point D (see **Fig. 3**).



**Fig. 5** Dependence of  $\beta_{theor}$  on ratio  $d_0/t_0$



$$\bar{\sigma}_{r,0} = \int_0^{\pi/2} \frac{\cos \alpha}{k + \sin \alpha} \cdot d\alpha = \ln \frac{k+1}{k} \quad (28.)$$

From the equation (28) the following results for  $\bar{\sigma}_{r,0} = 1$

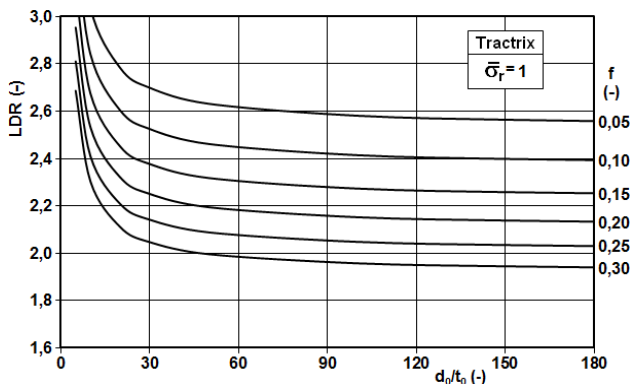
$$\frac{k+1}{k} = e \quad (29.)$$

The formula for evaluation of limiting drawing ratio  $\beta$  is expressed from the equation (14) what could be used to evaluate theoretical value of the LDR.

$$\beta_{\text{theor}} = \frac{k+1}{k} \cdot \left( 1 + \frac{2}{\frac{d_0}{t_0}} \right) \quad (30.)$$

where the expression  $(k+1)/k$  is substituted by the equation (29). Limit value of LDR is then determined according to the equation (30) for condition  $d_0/t_0 \rightarrow \infty$ .

$$\text{LDR} = \lim_{d_0/t_0 \rightarrow \infty} \beta_{\text{theor}} = \lim_{d_0/t_0 \rightarrow \infty} e \cdot \left( 1 + \frac{2}{\frac{d_0}{t_0}} \right) = e \cdot (1+0) = e \quad (31.)$$

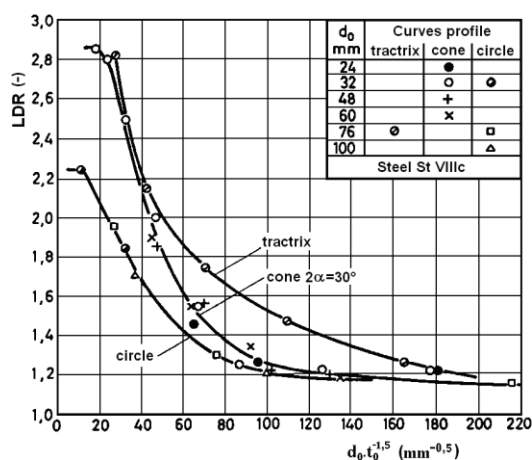


**Fig. 6** Dependence of LDR on  $d_0/t_0$  ratio and friction coefficient  $f$

This result is documented on graph in **Fig. 5**. The curve converges to value  $LDR=e$  with increasing of  $d_0/t_0$  ratio. The value of friction coefficient  $f$  is always  $f > 0$  in real application of deep drawing process. The visualization of differential equation (15) for friction coefficient  $f > 0$  and relative radial stress  $\bar{\sigma}_r = 1$  is shown in **Fig. 6**. Limit value of LDR is decreasing with the increase of friction coefficient  $f$ . High values of  $d_0/t_0$  ratio give the values of  $LDR < e$  and on the contrary, low values of  $d_0/t_0$  ratio ( $d_0/t_0 < 20$ ) give the values of  $LDR > e$ . In general, the limit value of LDR is the function of the  $d_0/t_0$  ratio and friction coefficient  $f$ .

### 3.3 Discussion

The definition of LDR shows that material has better ability to be drawn with increasing coefficient of friction. Many publications and examinations deal with that topic [17-20] to determine LDR limit. Various geometrical shapes of drawing die were examined such as are line, circle, parabola, sinusoid, ellipse and tractrix curve. In a view of minimal drawing force and maximal value of LDR, the best results were achieved with the use of the tractrix curve. This fact is documented in the publication of Shawki [9,21]. His results are shown in **Fig.7**. Limit value of LDR is expressed as a function of aggregated element  $d_0 \cdot t_0^{-1.5}$ . Maximal value of LDR reached limit value  $LDR = 2.85$ . The author of publication [7] did not define complete measurement conditions. The publication does not define for which value of LDR the curves were created. Marciniak et al. [22] indicated  $LDR = e \approx 2.72$  as the limit value. The value was determined differently than it is described in this paper. Morawiecki et al. [23] published **Table 1** where limit value achieves value of  $LDR=2.72$  what corresponds with the equation (31). Theoretical limit value is  $LDR = 2.25 \div 2.50$ . This value is reduced by the friction influence (the friction is not defined). Drastík and Elfmark [24] indicated the limit value of LDR as much as  $LDR=2.9$ . Theoretical analysis proves the possibility to reach such value as it is documented on graph in **Fig. 6**. The real measurements of LDR limit value prove the advisability of differential equation (15) application for relative radial stress calculation in the process of cupping without blank holder. Marumo [25] et al. and Sekhara-Reddy et al. [26] confirmed the dependency of LDR value on friction coefficient and also on depth of the material. Inoue and Takasui [27] documented the dependency of LDR value on the sheets anisotropy at rolled aluminum alloys sheets. Halkaci et al. [28] used finite element method (FEM) to calculate the LDR value.



**Fig. 7** Dependence of the  $LDR$  on  $d_0/t_0^{1.5}$  ratio and drawing die profile [9]

**Table 1** Limit values of LDR [23]

Name	Represented by symbol	Limit value	
		theoretical	technological
Limiting drawing ratio	LDR	2.72	2.25 – 2.50
Logarithmic limiting drawing ratio	$\ln(LDR)$	1.0	0.8 – 0.9

#### 4 Conclusions

It is necessary to describe measurement conditions for evaluating of LDR limit value of specific real material. The basic conditions are ratio of roundel (blank) diameter to cup diameter, friction coefficient and geometrical shape of cupping die. The differential equation (15) stated above properly describes state of the stresses along the high of cupping die in cupping process. Needed cupping force could be calculated based on stated relative radial stress. The paper documents the importance of friction coefficient for cupping test. If friction influence conditions are not defined, the verification of LDR limit value is problematic.

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### Nomenclature

$a$	– constant of tractrix curve [mm]
$C$	– integral constant [–]
$D_0$	– diameter of roundel (blank) [mm]
$d_0$	– inner diameter of the cup or the punch [mm]
$d_k$	– outer diameter of the cup [mm]
$e$	– Euler's number [–]
$f$	– friction coefficient [–]
$k$	– constant of differential equation $k=r_0/a$ [–]
$LDR$	– limiting drawing ratio [–]
$r$	– radius of the annular element [mm]
$r_0$	– inner radius of the cup or the punch [mm]
$r_k$	– outer radius of the cup [mm]
$t$	– wall thickness of the cup [mm]
$t_0$	– thickness of roundel (blank) [mm]
$\alpha$	– angle of element position [rad]
$\beta$	– limiting drawing ratio [–]
$\sigma_n$	– normal contact stress [MPa]
$\sigma_p$	– resistance to deformation [MPa]
$\sigma_r$	– radial stress [MPa]
$\bar{\sigma}_r$	– relative radial stress [–]
$\sigma_t$	– tangential stress [MPa]